Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2019

Subject Code: 2130002 **Date: 30/05/2019**

Subject Name:Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

MARKS

Q.1 **(a)** Solve $(x+y-2)\,dx+(x-y+4)\,dy=0$ **03**

(b) Solve $(1+y^2)+(x-e^{-\tan^{-1}y})\dfrac{dx}{dy}=0$ **04**

(c) Expand $f(x)=|\cos x|$ as a Fourier series in the interval $-\pi<x<\pi$ **07**

Q.2 **(a)** Define unit step function and unit impulse function. Also sketch the graphs. **03**

(b) Solve $\left(\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y\right)=4\sin 2x$ **04**

(c) Find the series solution of $y''+xy'+y=0$ about the ordinary point $x=0$. **07**

OR

(c) Find the Fourier series expansion for $f(x)$, if **07**

$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$, Also deduce that

$1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots\ldots=\dfrac{\pi^2}{8}$

Q.3 **(a)** Using Fourier integral representation, show that **03**

$$\int_0^\infty \frac{1-\cos \pi\omega}{\omega}\sin \omega x\, d\omega=\begin{cases}\dfrac{\pi}{2}, & 0<x<\pi\\ 0, & x>\pi\end{cases}$$

(b) Solve $\left(\dfrac{d^2y}{dx^2}+y\right)=x^2\sin 2x$ **04**

(c) Solve by method of variation of parameters $\left(\dfrac{d^2y}{dx^2}+9y\right)=\dfrac{1}{1+\sin 3x}$ **07**

OR

Q.3 **(a)** Find Laplace transform of $te^{at}\sin at$ **03**

(b) Solve $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=5e^x-\sin 2x$ **04**

(c) Solve

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$$ **07**

Q.4 **(a)** Find the orthogonal trajectories of the curve $y=x^2+c$ **03**

(b) Find the Laplace transform of (i) $\cos(at+b)$ **04**

(ii) $\sin^2 3t$

(c) State convolution theorem and apply it to evaluate **07**

$$L^{-1}\left(\frac{s^2}{\left(s^2+4\right)^2}\right)$$

OR

Q.4

(a) Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4y=0$ **03**

(b) Find Half range cosine series for $f(x)=(x-1)^2$ in the interval $0<x<1$ **04**

(c) Solve $y''+4y'+3y=e^{-t}, y(0)=y'(0)=1$ using Laplace transform. **07**

Q.5 **(a)** Form the partial differential equation by eliminating the arbitrary constants from $z=ax+by+a^2+b^2$ **03**

(b) Solve $(y-z)p+(x-y)q=z-x$ **04**

(c) Solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$, where $u(x,0)=4e^{-x}$ using the method of separation of variables. **07**

OR

Q.5 **(a)** Form the partial differential equation by eliminating the arbitrary function from $f(x^2+y^2, z-xy)=0$ **03**

(b) Solve $\log\left(\frac{\partial^2 z}{\partial x\partial y}\right)=x+y.$ **04**

(c) A bar with insulated sides is initially at temperature 0^0C, throughout. The end $x=0$ is kept at 0^0C and heat is suddenly applied at the end $x=l$ so that $\frac{\partial u}{\partial x}=A$ for $x=l$, where A is a constant. Find the temperature function. **07**

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